E0 232: PROBABILITY AND STATISTICS

PROBLEM SHEET 3
TUTORIAL ON 13^{TH} OCTOBER, 2016 (2.00 PM ONWARDS)
VENUE: CSA 117

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(1) **[Rohatgi 3.2.9,3.2.10]** $\lambda > 0$ is a constant, X is the Poisson RV with PMF $P\{X = x\} = \frac{\lambda^x}{x!} e^{-\lambda}$, $x = 0, 1, 2, \ldots$. Show / compute the following:

  (a) mean / 1^{st} moment, $\mu = E[X] = \lambda$
  (b) 2^{nd} moment, $E[X^2] = \lambda + \lambda^2$
  (c) 3^{rd} moment, $E[X^3] = \lambda + 3\lambda^2 + \lambda^3$
  (d) 4^{th} moment, $E[X^4] = \lambda + 7\lambda^2 + 6\lambda^3 + \lambda^4$
  (e) variance / 2^{nd} central moment, $Var(X) = \mu_2 = E[(X - \mu)^2] = \lambda$
  (f) 3^{rd} central moment, $\mu_3 = E[(X - \mu)^3] = \lambda$
  (g) 4^{th} central moment, $\mu_4 = E[(X - \mu)^4] = \lambda + 3\lambda^2$
  (h) coefficient of skewness, $\alpha_3 = \frac{\mu_3}{(\mu_2)^{3/2}} = ?$
  (i) kurtosis, $\alpha_4 = \frac{\mu_4}{(\mu_2)^2} = ?$

(2) **[Rohatgi 3.2.2]** From a box containing N identical tickets numbered 1 to N, n tickets are drawn with replacement. Let X be the largest number drawn. Find $E[X]$.

(3) **[Rohatgi 3.2.14]** Compute $E[X]$, $Var(X)$, and $E[X^n]$ ($n > 0$ integer) for the following PMF, where for $0 < p < 1$,
$P(X = x) = p(1 - p)^{x-1}, x = 1, 2, \ldots,$ and zero elsewhere.

(4) **[Ross 2.51]** A coin, having probability p of landing heads, is flipped until a head appears for the r^{th} time. Let N denote the number of flips required. Calculate $E[N]$.
Hint: There is an easy way of doing this. It involves writing N as the sum of r geometric RVs. Otherwise, recall one problem from previous problem sheet.

These problems have been taken from:

- Chapters 2 and 3 of *An Introduction to Probability and Statistics* by Rohatgi and Saleh, second edition.
- Chapter 2 of *Introduction to Probability Model* by Sheldon Ross, tenth edition.
- Chapter 1 of *An Exploration of Random Processes for Engineers* by Bruce Hajek, 2014.

Some of the problems have been modified slightly. The problems given here are not straightforward, and hence, solving other problems from suggested textbooks before trying these may be useful. Solutions of these problems will be discussed during the tutorial session.

(5) **[Ross 2.46]** If X is a nonnegative integer valued random variable, show that

$$E[X] = \sum_{n=1}^{\infty} P\{X \geq n\} = \sum_{n=0}^{\infty} P\{X > n\}$$

Hint: Define the sequence of RVs I_n, $n \geq 1$, as $I_n = 1$ if $X \geq n$, 0 otherwise. Now express X in terms of the I_n.

(6) **[Hajek 1.19]** Suppose the length L and width W of a rectangle are independent and each uniformly distributed over the interval $[0, 1]$. Let $C = 2L + 2W$ (the length of the perimeter) and $A = LW$ (the area).
(a) Find the means, variances, and probability densities of C and A.
(b) Solve the above problem when L and W are exponentially distributed with parameter $\lambda > 0$.

(7) **[Hajek 1.29]** Let the random variables X and Y be jointly uniformly distributed on the region $\{0 \leq u \leq 1, 0 \leq v \leq 1\} \cup \{-1 \leq u < 0, -1 \leq v < 0\}$.
(a) Determine the joint pdf f_{XY}.
(b) Find f_X, the marginal pdf of X.
(c) Find the conditional pdf of Y given that $X = a$, for $-1 \leq a \leq 1$.
(d) Find $E[Y|X = a]$ for $|a| \leq 1$.
(e) Are X and Y independent?
(f) What is the pdf of $Z = X + Y$?
(g) Find $\rho_{XY} = \dfrac{E[(X - EX)(Y - EY)]}{\sqrt{Var(X)Var(Y)}}$.
(ρ_{XY} is called the correlation coefficient of X and Y.)
You need not try (c) and (d) if conditional distribution and conditional expectation are not covered in class.

October 10, 2016; Dept. of CSA, IISc-Bangalore