
E0 203: SPECTRAL ALGORITHMS

ASSIGNMENT 1

(1) Let $A \in \mathbb{R}^{n \times n}$ symmetric matrix with eigenvalues $\lambda_1, \dots, \lambda_n$. Then show that

$$\text{trace}(A) = \lambda_1 + \dots + \lambda_n$$

(2) If A is a nonsingular matrix show that

$$\min_{\|X\|=1} \|Ax\| = \frac{1}{\|A^{-1}\|}$$

(3) For a graph G , with e edges and t triangles and adjacency matrix A , show that

- $(A^r)_{ij}$ is the number of paths between i and j of length r .
- $\text{tr}(A) = 0$.
- $\text{tr}(A^2) = 2e$.
- $\text{tr}(A^3) = 6t$.

(4) For a graph G , show that the multiplicity of zero in spectrum of Laplacian L is the number of connected components of G .

(5) For a graph G , show that the multiplicity of zero in spectrum of signless Laplacian $Q = D + A$, is the number of bipartite connected components of G .

(6) Let G be a graph with diameter d . Show that $A(G)$ and $L(G)$ have at least $(d + 1)$ distinct eigenvalues.

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