



# Deep Representation Learning for Prediction of Temporal Event sets in the Continuous Time Domain

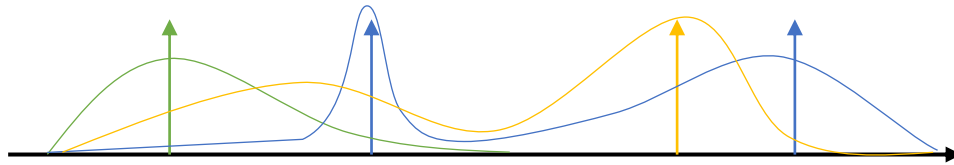
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Kawin is currently associated with Observe.ai and Pratyaksha with Goldman Sachs  
Work done during their graduate studies at IISc, Bangalore

# What are TPPs?

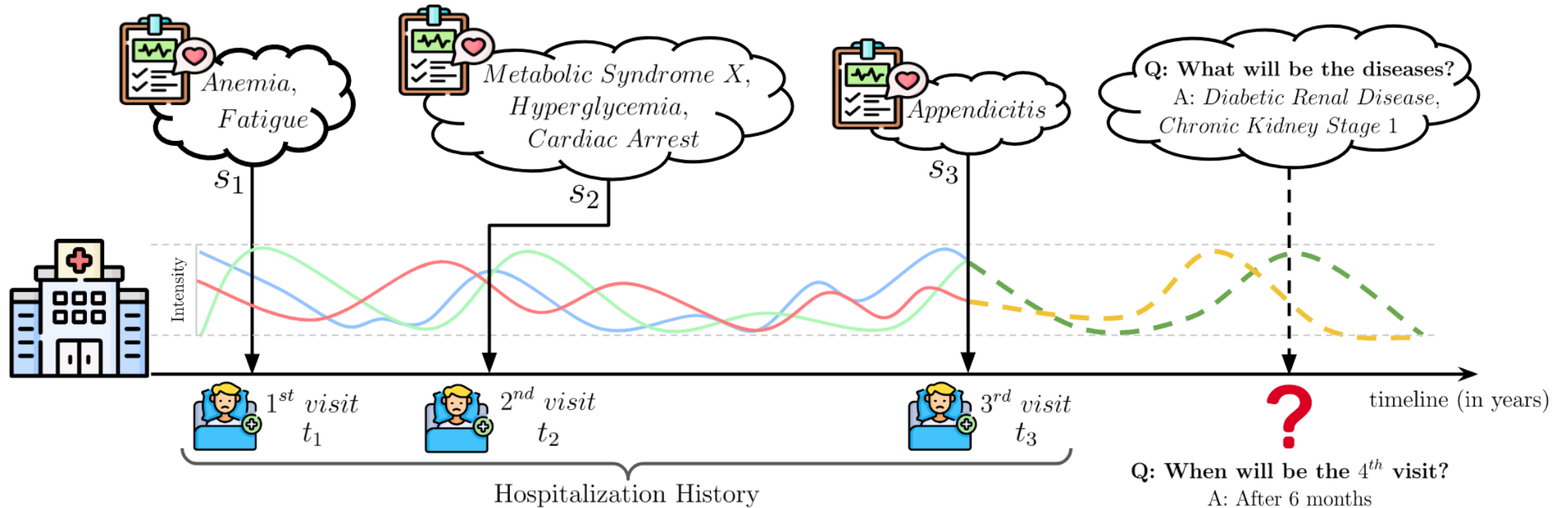


- Temporal Point Processes (TPPs) are probabilistic generative models for continuous-time event sequences.
- TPPs can be learned from data using traditional methods [1, 2, 3] and using deep learning [4, 5, 6].
- Some of the example use cases include modeling and predicting hospital visits, stock portfolio selection, and shopping basket checkouts.

1. M. Winkel – Poisson processes, generalizations and applications ([stats.ox.ac.uk/~winkel/bs3a07l1-3.pdf](https://stats.ox.ac.uk/~winkel/bs3a07l1-3.pdf))
2. T. Beckers – An introduction to Gaussian Process models ([arxiv.org/pdf/2102.05497.pdf](https://arxiv.org/pdf/2102.05497.pdf))
3. P. J. Laub and others – Hawkes Processes ([arxiv.org/pdf/1507.02822.pdf](https://arxiv.org/pdf/1507.02822.pdf))
4. Du and others – Recurrent Marked Temporal Point Processes ([arxiv.org/pdf/1705.05690.pdf](https://arxiv.org/pdf/1705.05690.pdf))
5. Mei and Eisner – The Neural Hawkes Process ([arxiv.org/pdf/1612.09328.pdf](https://arxiv.org/pdf/1612.09328.pdf))
6. Zuo and others – Transformer Hawkes Process ([arxiv.org/pdf/2002.09291.pdf](https://arxiv.org/pdf/2002.09291.pdf))

# Temporal Event Sets

The notion of sequence of events (in TPPs) is extended to a sequence of sets of events.



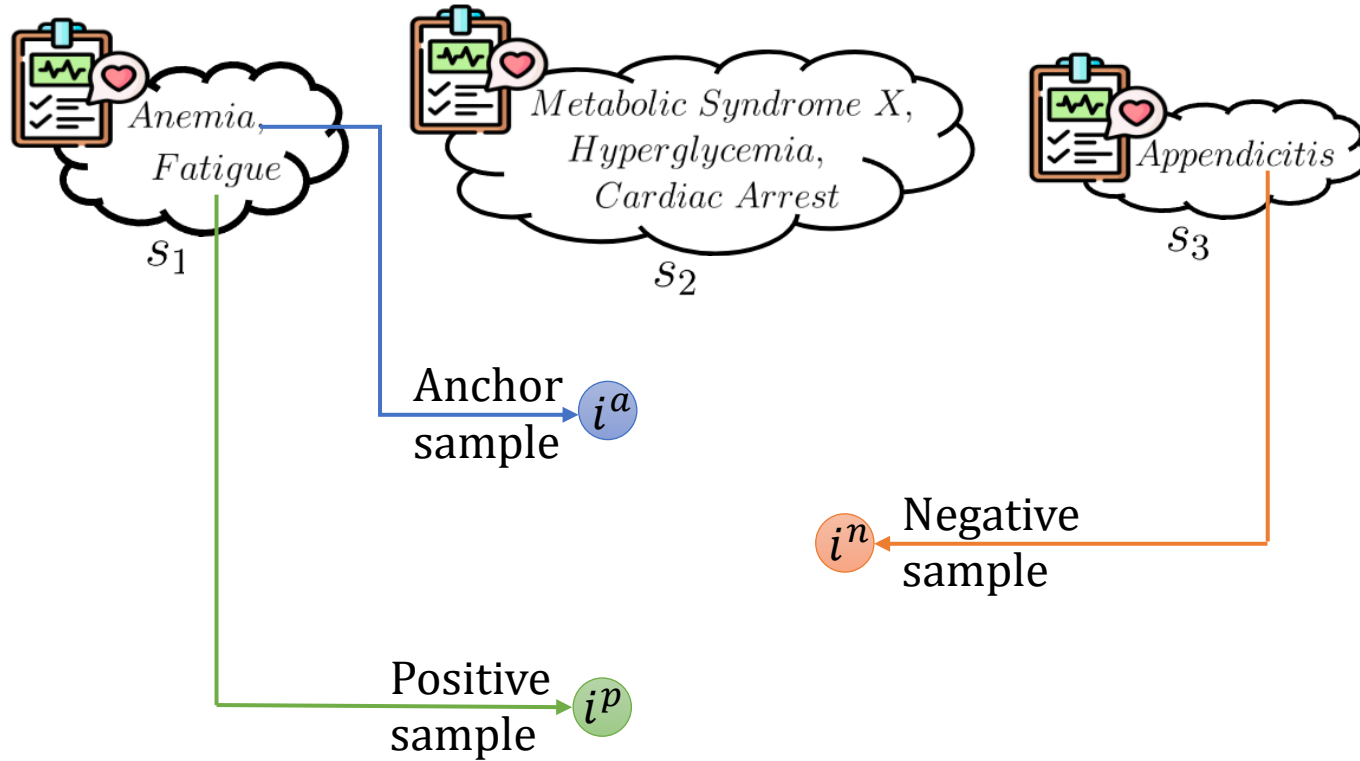
# Methodology

How to model Temporal Event sets?

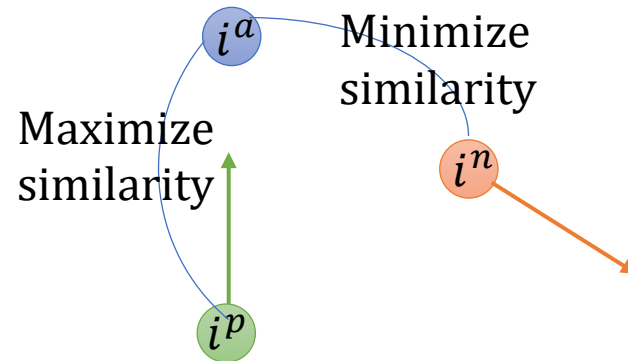
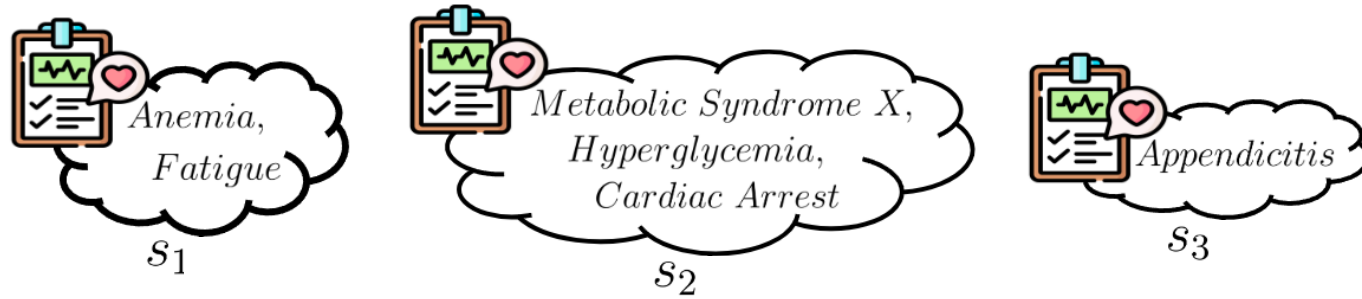
# Step 1

Learning Contextual Representations for items in event sets

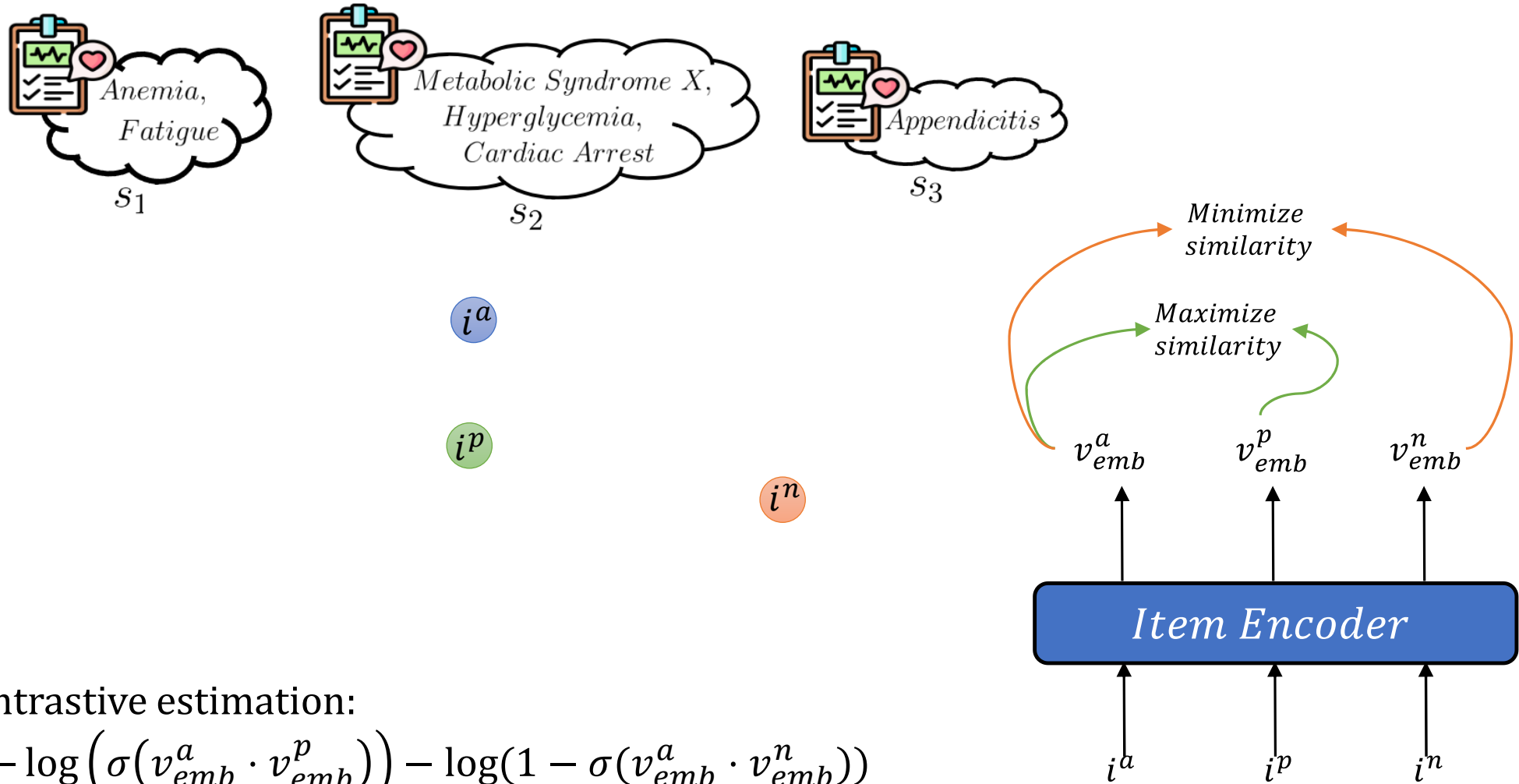
# Step 1: Contextual Item Embeddings



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Noise contrastive estimation:

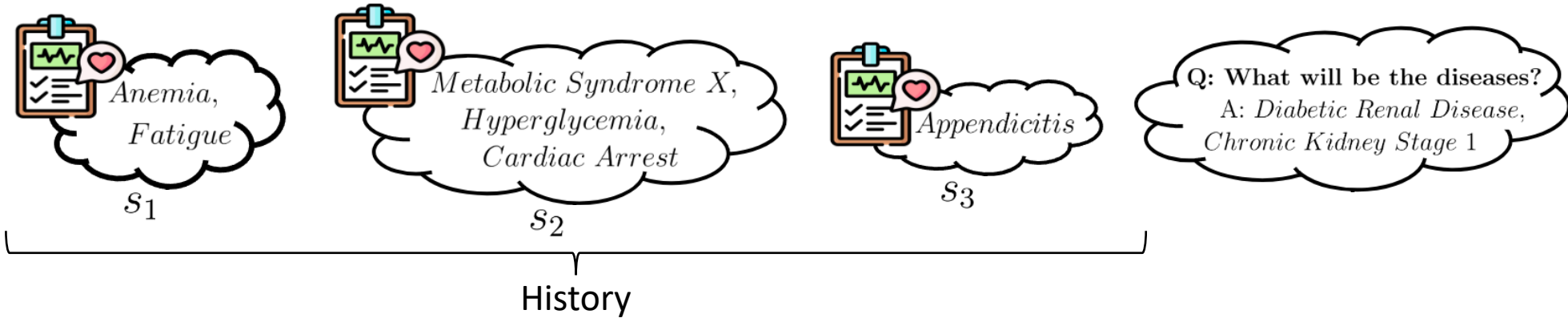
$$\mathcal{L}_{aux} = -\log(\sigma(v_{emb}^a \cdot v_{emb}^p)) - \log(1 - \sigma(v_{emb}^a \cdot v_{emb}^n))$$



# Step 2

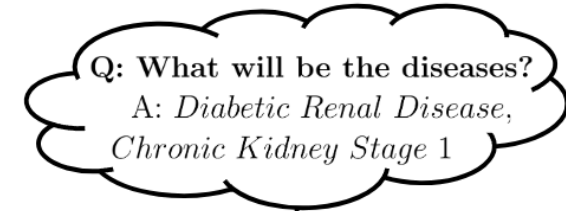
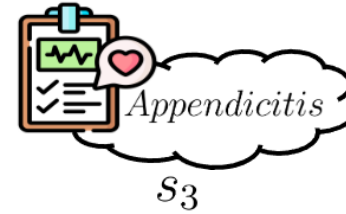
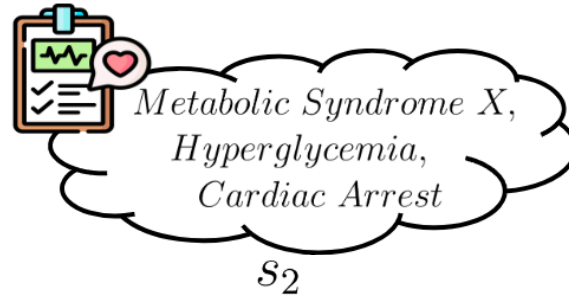
Modeling Temporal Event sets using the embeddings learnt in Step 1

# Step 2: Temporal Event set Modeling



Contains: event sets, corresponding timestamps, and associated domain specific features

# Step 2: Temporal Event set Modeling



$\mathcal{H}_4 = \{ \langle s_1, t_1, f_1 \rangle,$

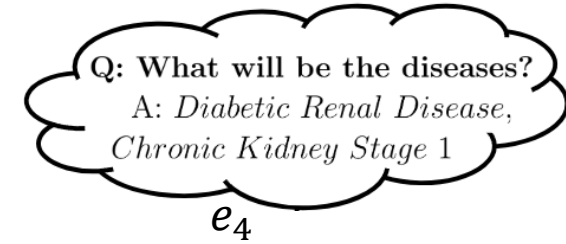
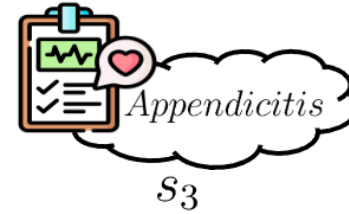
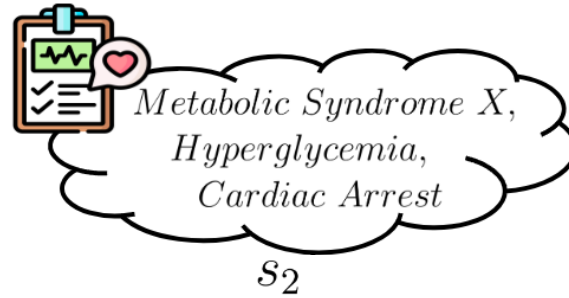
$\langle s_2, t_2, f_2 \rangle,$

$\langle s_3, t_3, f_3 \rangle$

}

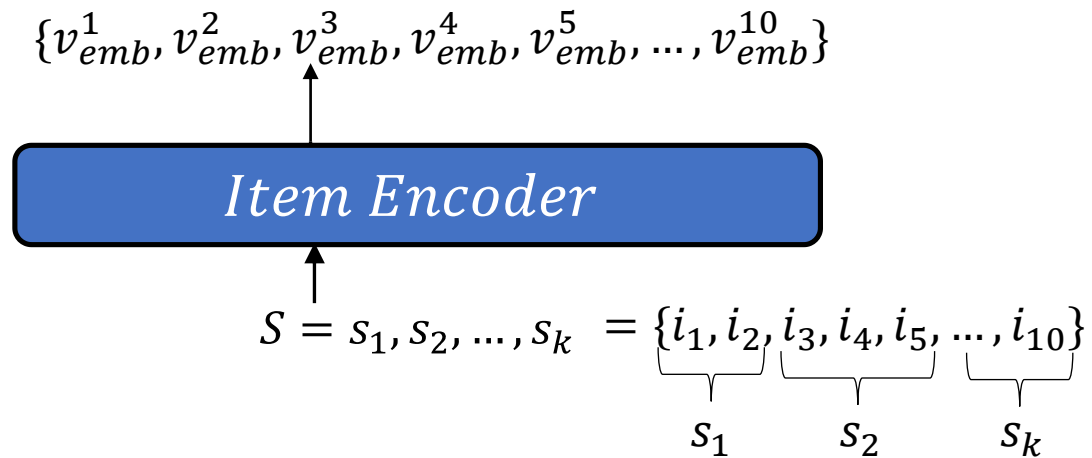
We need to predict:  $e_4$  and  $t_4$  given  $\mathcal{H}_4$   
 $e_4$  is a subset of the event set  
 $t_4$  is the time at which  $e_4$  occurred

# Step 2: Temporal Event set Modeling



$$\mathcal{H}_4 = \{ \langle s_1, t_1, f_1 \rangle, \langle s_2, t_2, f_2 \rangle, \langle s_3, t_3, f_3 \rangle \} \rightarrow \langle e_4, t_4 \rangle$$

# Step 2: Temporal Event set Modeling



# Step 2: Temporal Event set Modeling

$$\{ \underbrace{v_{emb}^1, v_{emb}^2}_{S_1}, \underbrace{v_{emb}^3, v_{emb}^4, v_{emb}^5}_{S_2}, \dots, \underbrace{v_{emb}^{10}}_{S_k} \}$$

Need to preserve the set relations (permutation invariance and equivariance)

But also differentiate among different different sets with how far they are from each other in the timeline

# Step 2: Temporal Event set Modeling

$$\mathbf{v}_{enc}^{pos}(j, d) = \begin{cases} \sin(j/10000^{\frac{2d}{d_{emb}}}) & ; \text{if } j \text{ is even} \\ \cos(j/10000^{\frac{2d}{d_{emb}}}) & ; \text{otherwise} \end{cases}$$

Hence, we introduce SpatioTemporal Encodings:

$$\mathbf{v}_{enc}^{temp}(\mathbf{t}_j, d) = \begin{cases} \sin(\mathbf{t}_j/10000^{\frac{2d}{d_{emb}}}) & ; \text{if } \mathbf{t}_j \text{ is even} \\ \cos(\mathbf{t}_j/10000^{\frac{2d}{d_{emb}}}) & ; \text{otherwise} \end{cases}$$

$$\mathbf{v}_{enc}(j, \mathbf{t}_j, d) = \mathbf{v}_{enc}^{pos}(j, d) + \mathbf{v}_{enc}^{temp}(\mathbf{t}_j, d)$$

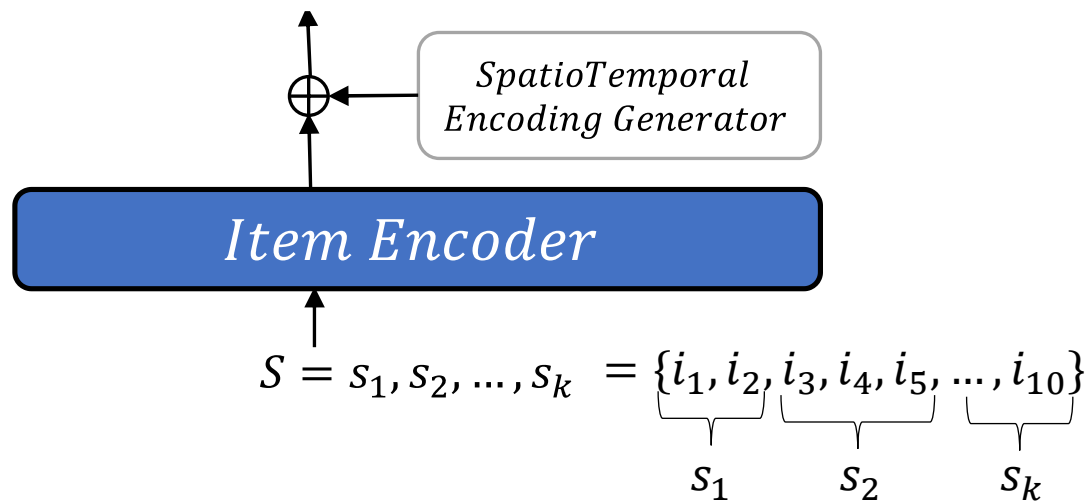
where  $\mathbf{t}_j$  is the timestamp corresponding to the  $j^{th}$  event set  $s_j$  for  $1 \leq j \leq k$

$$\underbrace{\{v_{emb}^1, v_{emb}^2\}}_{s_1}, \underbrace{\{v_{emb}^3, v_{emb}^4, v_{emb}^5\}}_{s_2}, \dots, \underbrace{\{v_{emb}^{10}\}}_{s_k}$$

Need to preserve the set relations (permutation invariance and equivariance)

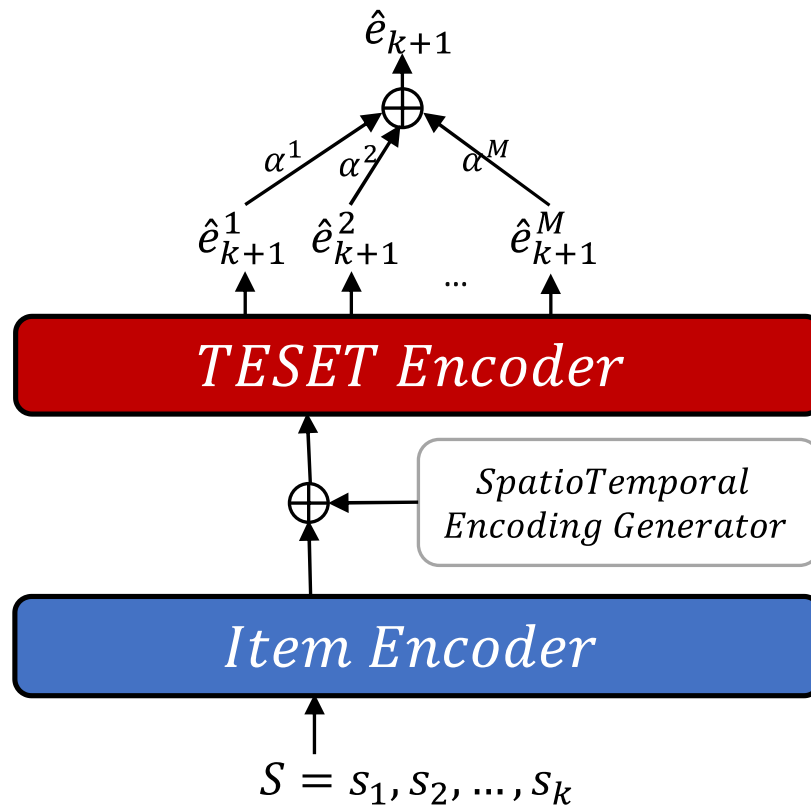
But also differentiate among different different sets with how far they are from each other in the timeline

# Step 2: Temporal Event set Modeling



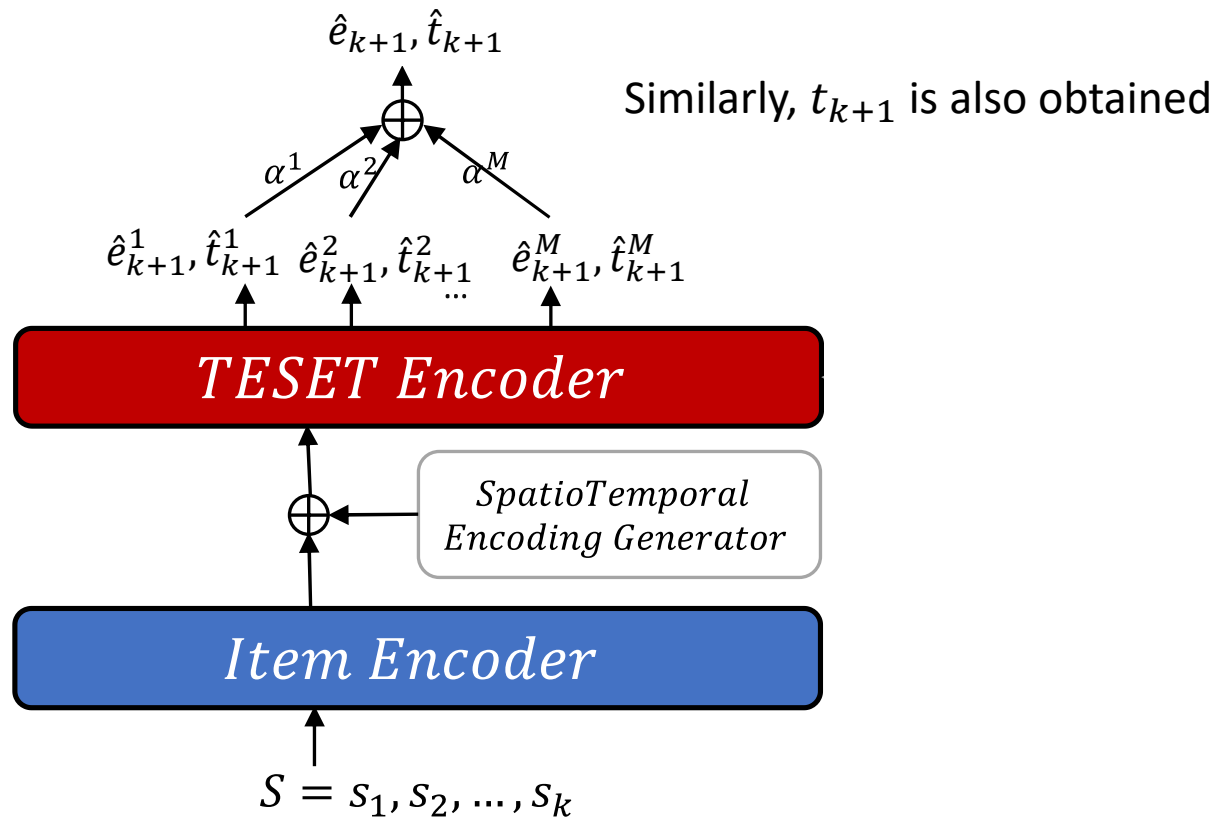


# Step 2: Temporal Event set Modeling

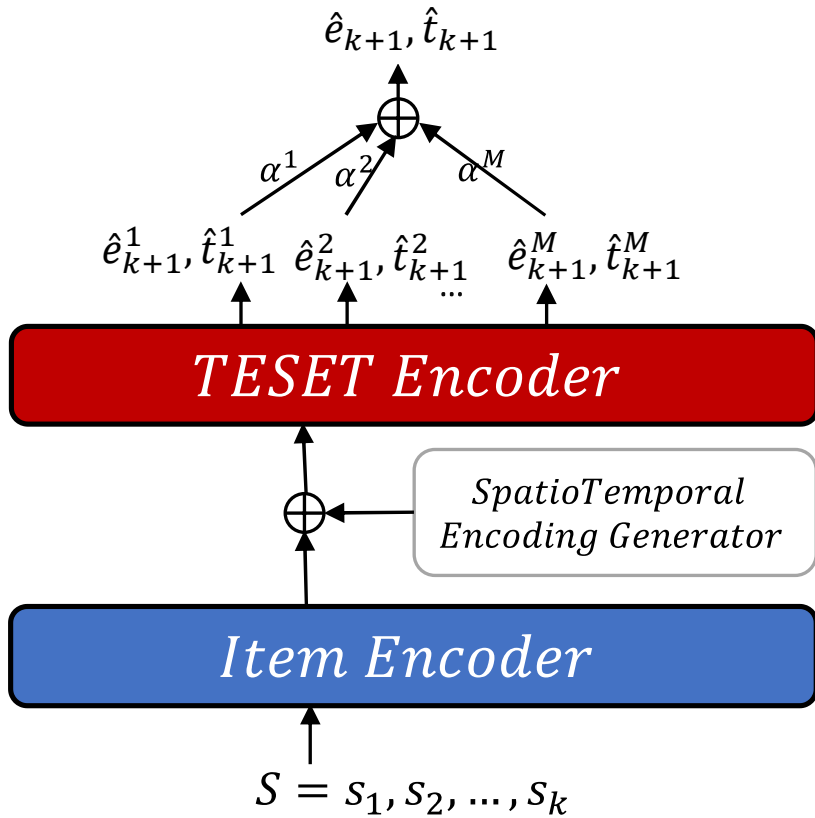


Each of the heads predicts a gaussian distribution corresponding to the intensity  
We use a mixture of gaussians to predict arbitrarily complex intensities

# Step 2: Temporal Event set Modeling



# Step 2: Temporal Event set Modeling



Losses ( $\mathcal{T}$  is the target set):

- *Binary Cross Entropy* (to model and predict event sets)

$$\mathcal{L}_{Event}^{BCE} = \frac{1}{|\mathcal{T}|} \sum_{d \in [|\mathcal{T}|]} \mathbb{1}_{\{\mathcal{T}^{(d)} \in \mathbf{e}_{k+1}\}} \hat{\mathbf{e}}_{k+1}^{(d)} + \mathbb{1}_{\{\mathcal{T}^{(d)} \notin \mathbf{e}_{k+1}\}} (1 - \hat{\mathbf{e}}_{k+1}^{(d)})$$

- *Dice Loss* (to handle class imbalance problem)

$$\mathcal{L}_{Event}^{Dice} = 1 - \frac{1}{|\mathcal{T}|} \sum_{d \in [|\mathcal{T}|]} \frac{2 \hat{\mathbf{e}}_{k+1}^{(d)} \mathbf{e}_{k+1}^{(d)} + \epsilon}{\sum_{d' \in [|\mathcal{T}|]} \hat{\mathbf{e}}_{k+1}^{(d')} + \mathbf{e}_{k+1}^{(d')} + \epsilon}$$

- *Huber Loss* (to learn temporal relations)

$$\mathcal{L}_{Temporal}^{Huber} = \begin{cases} \Delta^2/2 & ; \text{if } \Delta < \delta \\ \delta(\Delta - \delta/2) & ; \text{otherwise} \end{cases}$$

where  $\Delta$  is the absolute value of  $(\hat{t}_{k+1} - t_{k+1})$ .

We use a linear combination of the the above:

$$\mathcal{L} = \lambda_1 \mathcal{L}_{Event}^{BCE} + \lambda_2 \mathcal{L}_{Event}^{Dice} + \lambda_3 \mathcal{L}_{Temporal}^{Huber}$$

# Results

# Temporal Event set Modeling

Training method	Synthea		Instacart	
	Event set Predictions (DSC)	Time Predictions (MAE)	Event Set Predictions (DSC)	Time Predictions (MAE)
<i>Baselines:</i>				
Neural Hawkes Process	0.08	2.50	0.29	0.24
Transformer Hawkes Process	0.18	2.41	0.32	0.24
Hierarchical Model	0.12	2.51	0.30	0.23
<i>Ours:</i>				
Temporal Event Set Modeling	0.20	2.29	0.35	0.21
Temporal Event set Modeling + Contextual Embeddings	<b>0.30</b>	<b>2.17</b>	<b>0.42</b>	<b>0.18</b>

We gain improvement when compared to baselines irrespective of whether we use contextual embeddings

# Fine-tuning to down- stream tasks

Training method	Synthea		Instacart	
	Event set Prediction given time (DSC)	Time Prediction given event (MAE)	Event Set Prediction given time (DSC)	Time Prediction given event (MAE)
Trained from scratch				
Neural Hawkes Process	0.21	5.70	0.35	2.19
Transformer Hawkes Process	0.20	4.52	0.34	2.15
Hierarchical Model	0.19	5.29	0.34	2.20
Ours	<i>0.22</i>	<i>4.28</i>	<i>0.38</i>	<i>1.83</i>
Finetuned				
Neural Hawkes Process	0.13	6.01	0.30	2.29
Transformer Hawkes Process	0.19	4.60	0.33	2.24
Hierarchical Model	0.18	5.87	0.35	2.31
Ours	<b>0.25</b>	<b>3.91</b>	<b>0.41</b>	<b>1.19</b>

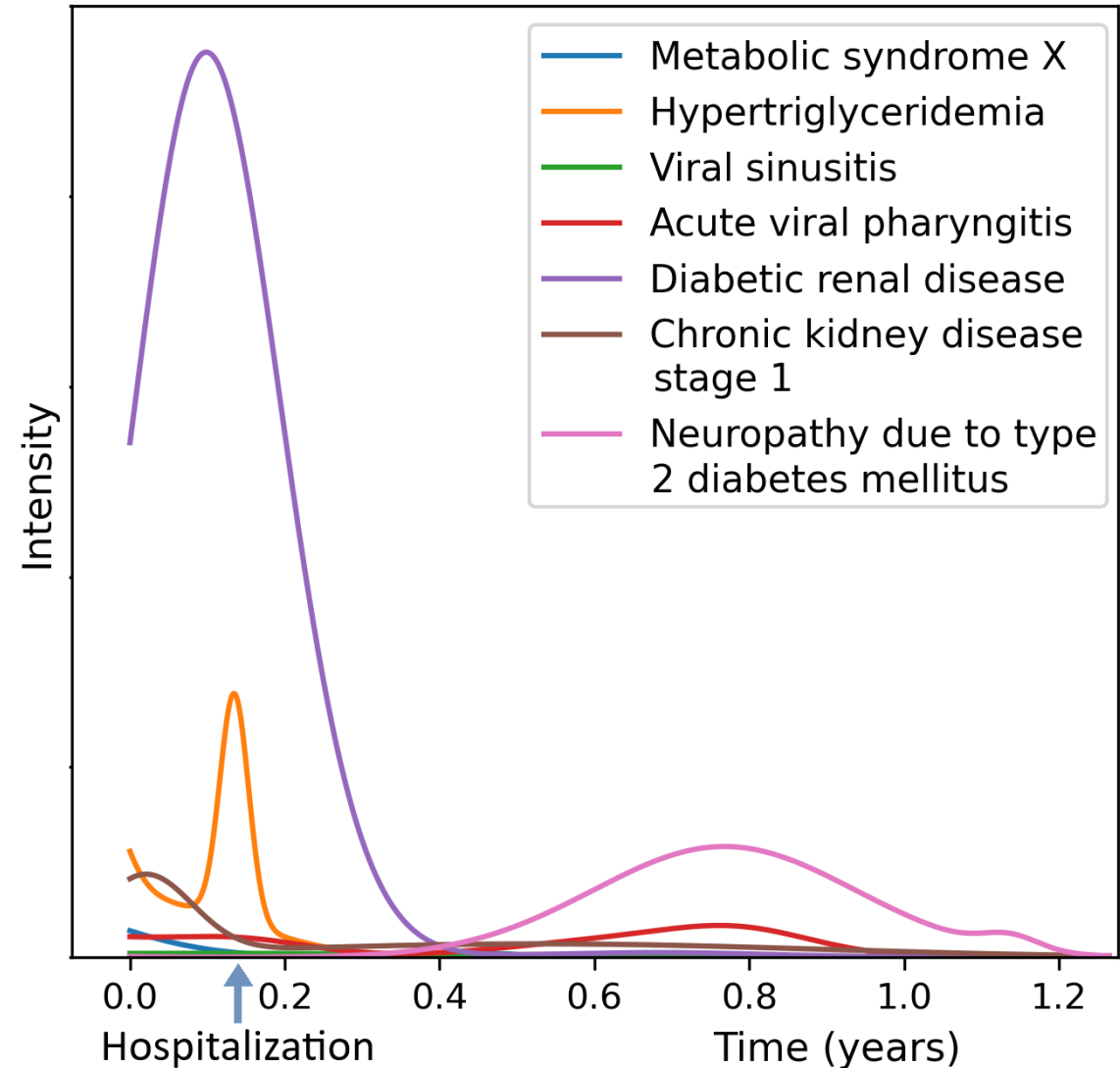
Note that the baselines are not good at being fine-tuned since training from scratch often gives better results.

Transfer Learning  
(from Synthea to MIMIC-III)

Training Method	Event Set Prediction given time (DSC)	Time Prediction given event (MAE)
Trained from scratch	0.47	0.70
Finetuned (from model pretrained on Synthea)	0.52	0.17

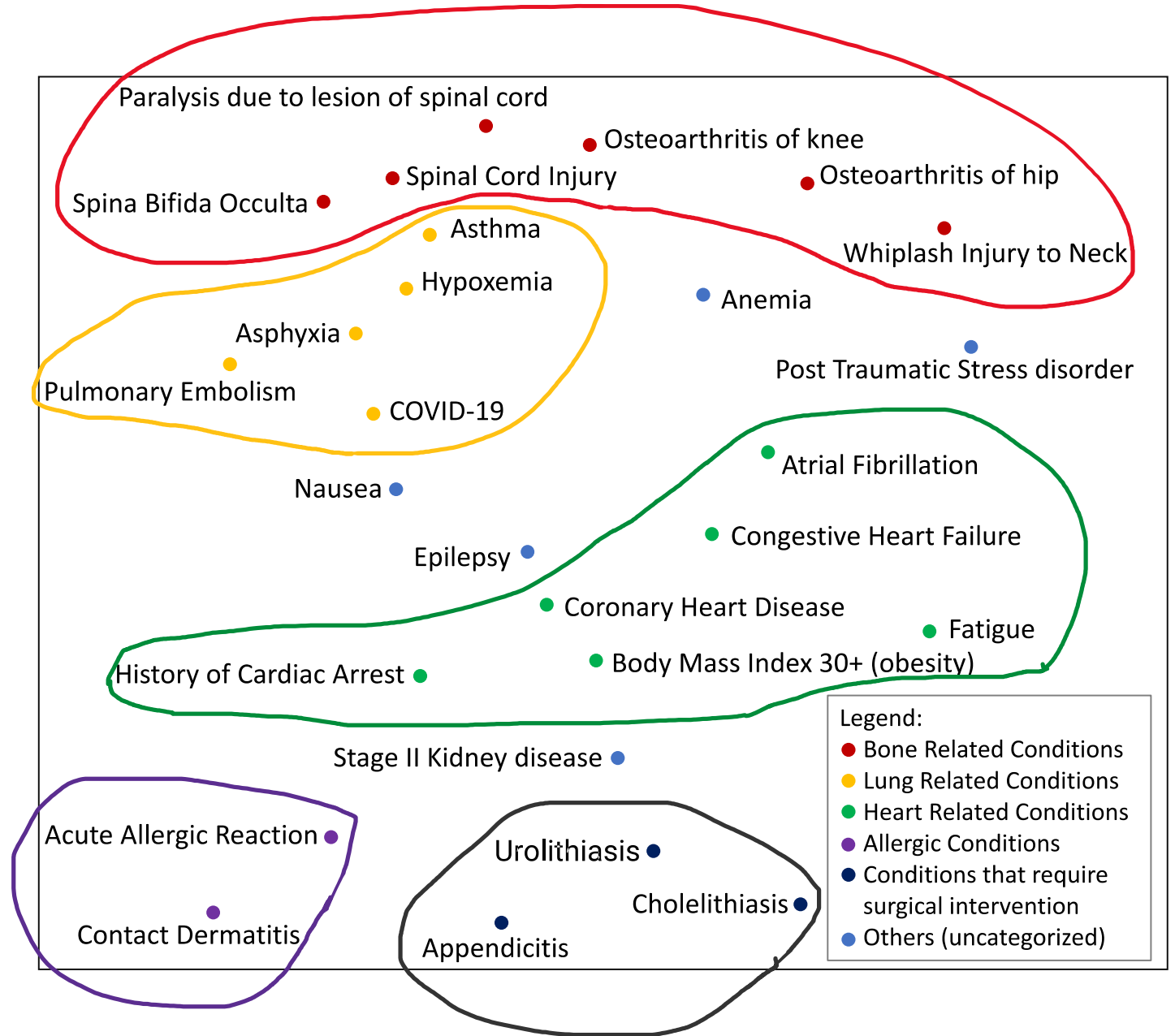
We give the sequence of hospitalization history of a female patient with a history of diabetes and the model predicts how the intensities of various diseases will evolve over time

Intensity  
Prediction  
Given  
History





# t-SNE of Contextual Event Embeddings



# Summary

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- We introduce Temporal Event set Modeling by extending TPPs.
  - We show a method to model the same using deep learning.
  - We empirically demonstrate the necessity of Temporal Event set Modeling by comparing to strong TPP based baselines.
  - We also try to understand the significance for each component in the algorithms through appropriate ablation experiments.
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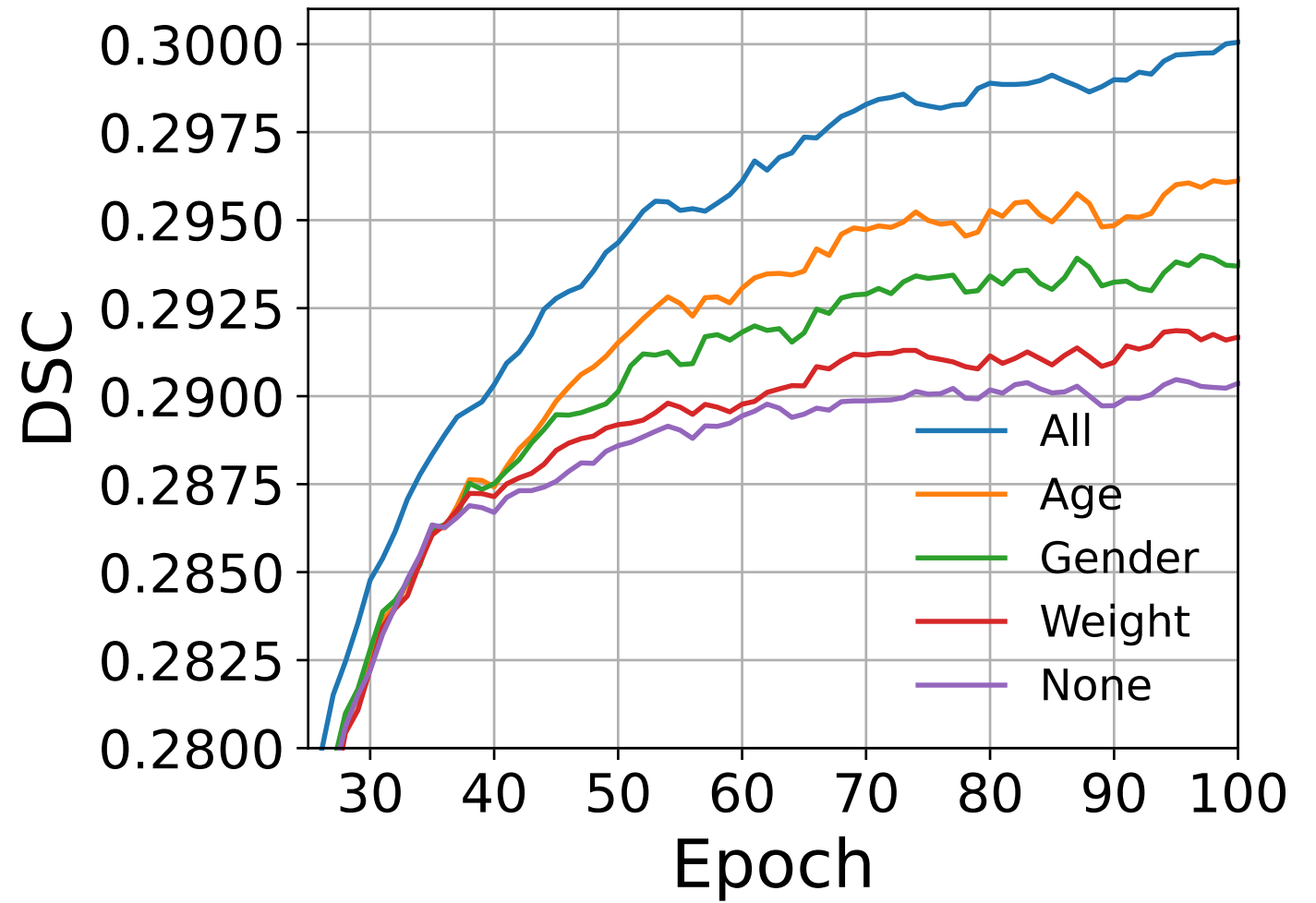
Thank You

Code available at: [https://github.com/paragduttaiisc/temporal\\_event\\_set\\_modeling](https://github.com/paragduttaiisc/temporal_event_set_modeling)

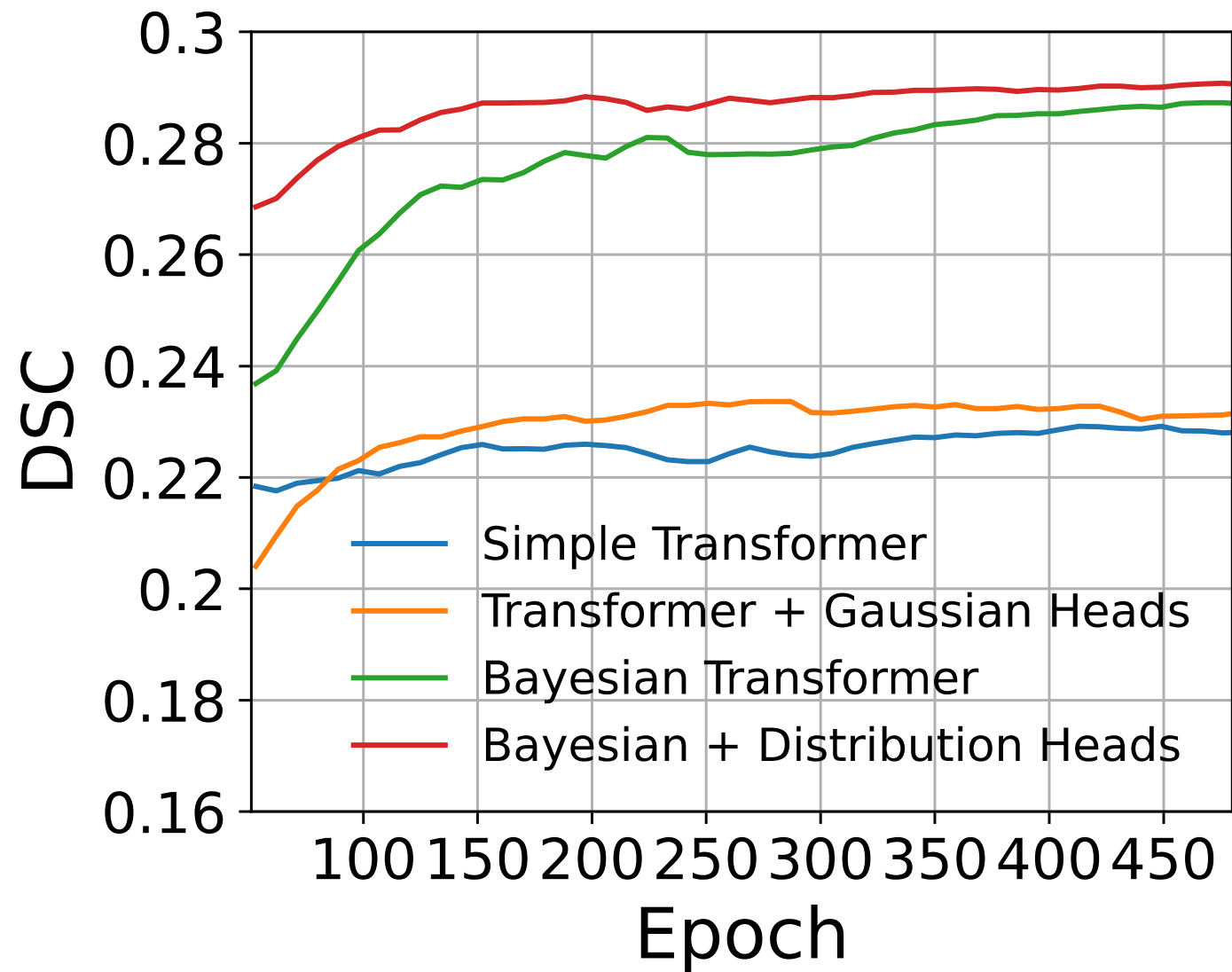


Additional Slides

Importance  
of Domain  
Specific  
Features



# Importance of Bayesian NNs



# Importance of Custom Encodings

Transformer Encodings	Event set Prediction (DSC)	Time Predication (MAE)
Positional Encodings	0.35	0.22
Ours	<b>0.42</b>	<b>0.18</b>

# Training Time and Compute Comparison

